RX-003-001616 Seat No. _____

B. Sc. (Sem. VI) (CBCS) Examination

March - 2019

Mathematics : MATH - 601(A)

(Graph Theory & Complex Analysis - 2)

(Old Course)

Faculty Code : 003
Subject Code : 001616

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are **compulsory**.
(2) Figures to the right indicate full marks of the question.

1 Answer the following questions briefly : **20**

(1) Define : Simple graph.
(2) Write the formula for total number of edges in a complete graph with n vertices.
(3) What is the degree of an isolated vertex in a graph ?
(4) Find the nullity of connected graph with 4 vertices and 8 edges.
(5) What is the number of pendant vertices in any binary tree with n vertices ?
(6) Define : Separable graph.
(7) Define : Self dual graph.
(8) Kuratowski's first graph K_5 has _____ vertices.
(9) What is the chromatic number of complete graph with 5 vertices ?
(10) Define : Acyclic diagraphs.
(11) Define : Power Series.
(12) Find radius of convergence for the series $\sum_{n=1}^{\infty} n!z^n$.
(13) Find fixed point of the bilinear transformation $W = \frac{3Z-4}{Z-1}$.

(14) Write the critical points of bilinear transformation $W = \frac{az+b}{cz+d}$.

(15) Write expansion of coshz in Maclaurian series.

(16) Write singular points of $\frac{\cos \pi z}{(z-1)(z-2)}$.

(17) Find residue of tanz at $Z = \frac{\pi}{2}$.

(18) Define : Residue of f(z) at pole Z_0.

(19) Define : Mobius mapping.

(20) Find Res $\left(\frac{\cos z}{z}, 0\right)$.

2 (a) Attempt any **three** : **6**

(1) Define : Pendant vertex, Complete graph.

(2) State and prove first theorem of Graph theory.

(3) Obtain the number of internal vertices in a binary tree with n vertices.

(4) Define : minimal dominating set, domination numbers.

(5) How many vertices and edges in Kuratowski's second graph $K_{3,3}$ has ?

(6) Define : Path matrix.

(b) Attempt any **three** : **9**

(1) What is the number of vertices in the complete graph K_n if it has 45 edges ?

(2) Prove that A graph is a tree iff it is minimally connected.

(3) Prove that the number of vertices n in a binary tree is always odd.

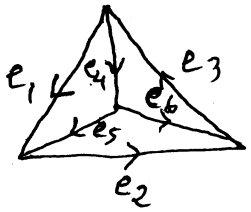
(4) In any simple connected planar graph with f regions, n vertices and e edges $(e > 2)$ then prove that

(i) $e \geq \frac{3}{2} f$

(ii) $e \leq 3n - 6$

(5) Prove that every tree with two or more vertices is 2-chromatic.

(6) Find minimal decyclization for the following graph :



(c) Attempt any **two** : **10**

(1) Explain Konigsberg bridge problem and the solution given by Euler.

(2) State and prove necessary and sufficient conditions for a graph to be an Euler graph.

(3) Prove that a connected plannar graph with n vertices and e-edges has e–n+2 regions.

(4) Define : Adjacency matrix and state its properties.

(5) Prove that the complete graph of five vertices is non-planner.

3 (a) Attempt any **three** : **6**

(1) Find region of convergence and radius of convergence for the series $\sum_{n=1}^{\infty} \frac{Z^n}{7^n + 1}$.

(2) Expand sinz in Taylor's series for Z_0=0.

(3) Evaluate $\int_c \frac{2Z + 3}{z(Z - 1)} dz$ where $c : |z| = 2$.

(4) Discuss for finding residue of $f(Z)$ at simple pole Z_0.

(5) Find the image of $|Z + 1| = 1$ under the mapping $W = \frac{1}{Z}$.

(6) Show that $x + y = 2$ transform into the parabola $u^2 = -8(v - 2)$ under the transformation $W = Z^2$.

(b) Attempt any **three** : **9**

(1) If $0 < |Z| < 4$ then prove that $\frac{1}{4Z - Z^2} = \sum_{n=0}^{\infty} \frac{Z^{n-1}}{4^{n+1}}$.

(2) Prove that $\frac{1}{Z^2} = \frac{1}{4} + \frac{1}{4} \sum_{n=1}^{\infty} (-1)^n (n+1) \left(\frac{Z-2}{2} \right)^n$.

(3) Find the image of the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $W = \frac{1}{Z}$.

(4) Prove that the transformation $W = 2Z + Z^2$ maps the unit circle $|Z| = 1$ of Z-Plane into a cardiode in W-plane.

(5) Prove that $\underset{z=i}{\text{Res}} \frac{Z^{1/2}}{\left(Z^2+1\right)^2} = \frac{1-i}{8\sqrt{2}}$ where $|Z| > 0, 0 < \arg z < 2\pi$.

(6) Find the value of integral $\int_C \frac{dz}{Z^3 (Z+4)}$ where $C : |Z| = 2$.

(c) Attempt any **two** : **10**

(1) State and prove Taylor's infinite series for an analytic function.

(2) Show that composition of two bilinear transformation is a gain a bilinear transformation.

(3) Discuss the bilinear mapping $W = Z^2$.

(4) State and prove Cauchy's residue theorem.

(5) Prove by residue theorem.

$$\int_0^{\infty} \frac{x \sin ax}{\left(x^2 + k^2\right)} dx = \frac{\pi}{2} e^{-ak} \quad (a > 0, k > 0)$$